Probability

Class Activity 5

Due: Wednesday at noon

Name:_____

Definition: Recall that the conditional probability of an event A given that event B has occurred is given by

$$P(A|B) =\_\_\_\_\_$$

and as a consequence we have

$$P(A \text{ and } B) =\_\_\_\_\_$$

Definition: Two events are **independent** if

$$P(A|B) =\_\_\_\_\_,$$

so knowing event B has no bearing on the probability of A.

Definition: Two events are **mutually exclusive** if

$$P(A \text{ and } B) =\_\_\_\_\_,$$

so both events cannot happen at the same time.

Example: Are the events of rolling an even number and drawing a red card from a deck independent? What is the probability of rolling an even number and drawing a red card?

Example: Five people choose a number at random from 1 to 10. The outcome of interest is the five chosen numbers. How many possible outcomes are there? How many ways can the 5 people choose all different numbers? What is the probability that 5 people all choose different numbers?

Example: What is the probability that in a room of thirty people (you included), what is the probability that someone else has your birthday?

Example: What is the probability that in a room of thirty people (you included), what is the probability that two people have the same birthday?

Example: You are buying a used car in city where rainfall causes street flooding often. You know that 5% of used cars have been damaged from flooding and 80% of those cars will later experience serious engine problems. On the other hand, only 10% of cars without flood damage will experience the same engine issues. What is the probability the car you buy will later experience engine issues?

1. Of 100 people surveyed 43 men were right handed while 9 were left handed. 44 women were right handed while 4 were left handed. What is the probability that a person was left-handed given that they are male? Are the events $L = \{\text{left handed}\}$ and $M = \{\text{male}\}$ independent?

2. You have a box with two balls in it, one red and one blue. We select one ball from the box, put it back and select another.
   a. Let's say RR is the event where you get the red ball twice, what is $P(RR)$?

   b. Let's say F is the event that you get the red ball on your first pull, what is $P(RR|F)$?

   c. Are the two events independent? Are they mutually exclusive? Justify your answer.

3. On a given day the probability that I go to a coffee shop is $P(CS) = .2$, the probability that I play chess is $P(Chess) = .5$, and $P(Chess|CS) = .8$. Find the probability $P(CS \text{ and } Chess)$.

4. Recall the situation where you are drawing two marbles from a bag with two red marbles, two blue, and one green. Draw a tree diagram that represents the situation.

5. Use your tree diagram to answer the following:
   a. What is the probability of drawing two red?

   b. What is the probability of drawing two blue?

6. Consider the following game where there are three dice with sides:

$$\text{Die } A : \{1, 1, 5, 5, 5, 5\}$$
$$\text{Die } B : \{3, 3, 3, 4, 4, 4\}$$
$$\text{Die } C : \{2, 2, 2, 2, 6, 6\}$$

The game is as follows: two players take turns selecting a die and whoever rolls the highest number wins.

a. What is the probability that Die A beats Die B?

b. What is the probability that Die B beats Die C?

c. What is the probability that Die C beats Die A?

d. What can you do to maximize your odds of winning the game?