# More Probability

**Example:** You are in a room with 5 other people. Assuming every birthday is equally likely and excluding leap years, what is the probability that someone has the same birthday as you?

**Example:** You are in a room with 5 other people. Assuming every birthday is equally likely and excluding leap years, what is the probability that two people have the same birthday?

**Definition:** The **complement** of an event $A$ is given by

$$P(A^c) = \_\_\_\_\_$$

**Definition:** The **conditional probability** of an event $A$ given that event $B$ has occurred is given by

$$P(A|B) = \_\_\_\_\_$$

and as a consequence we have

$$P(A \text{ and } B) = \_\_\_\_\_$$

**Definition:** Two events are **independent** if

$$P(A|B) = \_\_\_\_\_,$$

so knowing event $B$ has no bearing on the probability of $A$. As a consequence we have

$$P(A \text{ and } B) = \_\_\_\_\_$$

**Definition:** Two events are **mutually exclusive** if

$$P(A \text{ and } B) = \_\_\_\_\_,$$

so both events cannot happen at the same time.

**Example:** Are the events of rolling an even number and drawing a red card from a deck independent? What is the probability of rolling an even number and drawing a red card?

**Example:** You are buying a used car in city where rainfall causes street flooding often. You know that 5% of used cars have been damaged from flooding and 80% of those cars will later experience serious engine problems. On the other hand, only 10% of cars without flood damage will experience the same engine issues. What is the probability the car you buy will later experience engine issues?

1. Five people choose a number at random from 1 to 10. The outcome of interest is the five chosen numbers. How many possible outcomes are there? How many ways can the 5 people choose all different numbers? What is the probability that 5 people all choose different numbers?

2. Of 100 people surveyed 43 men were right handed while 9 were left handed. 44 women were right handed while 4 were left handed. What is the probability that a person was left-handed given that they are male? Are the events $L = \{\text{left handed}\}$ and $M = \{\text{male}\}$ independent?

3. You have a box with two balls in it, one red and one blue. We select one ball from the box, put it back and select another.

   a. Let's say $RR$ is the event where you get the red ball twice, what is $P(RR)$?

   b. Let's say $F$ is the event that you get the red ball on your first pull, what is $P(RR|F)$?

   c. Are the two events independent? Are they mutually exclusive? Justify your answer.

4. On a given day the probability that I go to a coffee shop is $P(CS) = .2$, the probability that I play chess is $P(Chess) = .5$, and $P(Chess|CS) = .8$. Find the probability $P(CS \text{ and } Chess)$.

5. Use your tree diagram to answer the following:

   a. What is the probability of drawing two red?

   b. What is the probability of drawing two blue?

6. Consider the following game where there are three dice with sides:

$$\text{Die } A : \{1, 1, 5, 5, 5, 5\}$$
$$\text{Die } B : \{3, 3, 3, 4, 4, 4\}$$
$$\text{Die } C : \{2, 2, 2, 2, 6, 6\}$$

The game is as follows: two players take turns selecting a die and whoever rolls the highest number wins.

a. What is the probability that Die $A$ beats Die $B$?

b. What is the probability that Die $B$ beats Die $C$?

c. What is the probability that Die $C$ beats Die $A$?

d. What can you do to maximize your odds of winning the game?