

1. Among various ethnic groups, the standard deviation of heights is known to be approximately three inches. We wish to construct a 95% confidence interval for the mean height of male Swedes. Forty-eight male Swedes are surveyed and the following data is collected.

	age (yr.)	height (in.)	weight (kg.)
average	34	71	82
sd.	5	2.8	10

- (a.) List the values relevant to calculating the confidence interval of interest:
- $\bar{x} =$
 - $\sigma =$
 - $n =$
- (b.) Describe the random variables X and \bar{X} in words.
- (c.) Explain why the conditions for inference are satisfied.
- (d.) Construct a 95% confidence interval for the population mean height of male Swedes and state your results within the context of the problem.

2. Political scientists are interested in knowing the population percent of voters who do not identify with a particular political party.
- (a.) When designing a study to determine this population proportion, what is the minimum number of respondents they would need to survey to be 95% confident that the population proportion is estimated to within 0.05?
- (b.) If it were later determined that it was important to be 98% confident and a new survey was done, how would that affect the minimum number of people they would need to survey? Explain your reasoning?
- (c.) The group of political scientists receive a large injection of grant money and can now perform a large survey of 5000 people. They perform a simple random sample of this size and found that 400 voters do not identify with a political party. We are interested in constructing a confidence interval for the true proportion of voters in this group.
- (a.) List the values relevant to calculating the confidence interval of interest:
- $\hat{p} =$
 - $n =$
- (b.) Describe the random variables X and \hat{P} in words.
- (c.) Explain why the conditions for inference are satisfied.
- (d.) Construct a 95% confidence interval and state your results within the context of the problem.
- (e.) The political scientists will spend money trying to recruit people that do not have a party if they have reason to believe that this group makes up more than 10% of the voting population. Based on the results above, should they pursue this group? Explain your reasoning.

3. Construct a confidence interval for each of the following scenarios (assume conditions for inference are satisfied):

a. $\bar{x} = 15$, $s = 5.5$, $n = 20$, $\alpha = .1$

b. $\hat{p} = .15$, $n = 20$, $\alpha = .1$

c. $\bar{x} = 15$, $\sigma = 5.5$, $n = 20$, $\alpha = .1$

d. $\bar{x} = 15$, $\sigma = 5.5$, $n = 20$, $\alpha = .05$

e. $\bar{x} = 15$, $\sigma = 5.5$, $n = 40$, $\alpha = .1$

4. Draw a picture of the given distribution with appropriate area shaded and find the probability:

a. $\mu = 15$, $\sigma = 5.5$, $n = 20$, $P(14.5 < \bar{X} < 15.5)$

b. $\mu = 15$, $\sigma = 5.5$, $n = 20$, $P(14.5 \leq \bar{X} \leq 15.5)$

c. $\mu = 15$, $\sigma = 5.5$, $n = 50$, $P(14.5 \leq \bar{X} \leq 15.5)$

d. $\mu = 15$, $\sigma = 5.5$, $n = 50$, $P(14.7 \leq \bar{X} \leq 14.9)$

e. $p = .15$, $n = 50$, $P(\hat{P} < .151)$

f. $p = .55$, $n = 50$, $P(.55 < \hat{P} < .58)$

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5. The print on the package of 100-watt General Electric soft-white lightbulbs claims that these bulbs have an average life of 750 hours. Assume that the lives of all such bulbs have a normal distribution with a mean of 750 hours and a standard deviation of 55 hours. Let \bar{x} be the mean life of a random sample of 25 such lightbulbs. Find the mean and standard deviation of the sample mean \bar{x} and describe the shape of its sampling distribution.
6. The management at a bank does not want its customers to wait in line for service for too long. The manager of a branch of this bank estimated that the customers currently have to wait for an average of 8 minutes for service. Assume that the waiting times for all customers at this branch have a normal distribution with a mean of 8 minutes and a standard deviation of 2 minutes.
- a. Find the probability that a randomly selected customer will have to wait for less than 3 minutes.
- b. What percentage of the customers have to wait for between 10 and 13 minutes?
- c. What percentage of the customers have to wait for 6 to 12 minutes?
- d. A customer claims to have waited for longer than 16 minutes and demands benefits on their account for the poor service. Does it seem likely that the customer is telling the truth? Explain your reasoning.

Throughout, X is a random variable and x_i is a particular value of X .

$$\mu = \frac{\sum x_i}{N}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$\bar{X} = \frac{\sum x_i}{n}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\sigma = \sqrt{\frac{\sum(\mu - x_i)^2}{N}}$$

$$E(X) = \sum x_i P(X = x_i)$$

$$s = \sqrt{\frac{\sum(\bar{X} - x_i)^2}{n-1}}$$

$$\sigma(E(X)) = \sqrt{\sum (x_i - E(X))^2 P(X = x_i)}$$

$$IQR = Q_3 - Q_1$$

$$\text{whiskers} = \begin{cases} Q_1 - 1.5 \cdot IQR \\ Q_3 + 1.5 \cdot IQR \end{cases}$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$\left(\frac{\bar{x} - \mu}{s/\sqrt{n}}\right) \sim t(n-1)$$

$$z\text{-score} = \frac{\text{observation} - \text{expected}}{\text{standard deviation}}$$

$$\hat{P} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

$$t\text{-score} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

A t -table shows the t -score corresponding to a certain confidence level and degrees of freedom. For example, 1.119 is in column $t_{.85}$ and row with degrees of freedom 7. This means that the t -score corresponding to the 85th percentile on the t -distribution with 7 degrees of freedom is 1.119.

t Table

cum. prob one-tail two-tails	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

A z -score table shows the percentage of values (usually a decimal figure) to the left of a given z -score on a standard normal distribution. For negative z -scores, look up the positive version on this table, and subtract it from 1.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998